

How Not to Miss a Missing Premise and How to Miss a Not-Missing One: The Case of Lewis Carroll's Tortoise*

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1. In Lewis Carroll's (1895) parable, Achilles tries to persuade the tortoise to accept the following argument [a₁]:

A: Things that are equal to the same are equal to each other.

B: The two sides of this triangle are things that are equal to the same.

Z: So, the two sides of this triangle are equal to each other.

Achilles fails because he encounters an infinite progression of hidden premises of the form "If all the premises of the argument are true, the conclusion is true". In [a₁], the hidden premise is H₁ "If A and B then Z" – surely, if one did not believe that H₁ is true, one would have a reason not to accept the conclusion Z. So, the argument [a₂] must lead to conclusion Z from A, B and H₁. But, one will have to supplement [a₂] with H₂: "If A and B and H₁ then Z" since if one did not believe H₂ one would have a reason not to draw conclusion Z. And so on ad infinitum.

The puzzle can be seen as arising through the application of an apparently innocent principle of discerning missing premises (§2). If looked at in this light, the standard responses given to the paradox do not so much resolve the puzzle as legislate against it being raised with respect to principles of inference (§3). I argue that a fundamental ambiguity infests the test for

what is a missing premise (§4). Moreover, the presence of the ambiguity explains why the puzzle appears to, though it does not, arise (§5). I end with some comments on the usefulness of the test (§6).

2. What is necessary to generate this infinite regress is the reasoning that establishes that H_1 , H_2 , ... are missing premises.

What does it mean to say that a statement is a missing premise in argument? Roughly, a premise is missing if (a) it is a premise of the argument (the conclusion follows when it is added to the existing premises) and (b) there would be no reason to believe the conclusion unless one believed that premise. Condition (a) is relatively clear. The real question concerns (b). Let us begin with the following approximation:

(*T*) If one did not believe P_M , one would have no reason to believe that C is true.

Consider the argument: "Killing is wrong, therefore abortion is wrong". The claim that abortion is a killing is the missing premise here because if one did not accept the claim, if one believed that abortion is not a killing, one would have no reason to believe that abortion is wrong.

If the tortoise accepted A and B but believed also that Z does not follow from them (i.e. did not accept H_1), he would certainly (we might think) have a reason for not drawing the conclusion Z. So, to the extent that he is willing to draw the conclusion it must be because he also believes that H_1 is true. And H_2 . And so on. Or so it might seem.

3. This puzzle is not removed by mentioning that there is a distinction between principles of inference and statements (e.g. Brown 1954). It does, of course, apply to principles of inference (which is the reason why it is troubling) but it does so by appealing to an apparently innocuous principle of discerning missing premises. And that principle is quite general and certainly not

geared specifically (at any rate) toward codifying principles of inference into statements. The surprising point of Carroll's puzzle is rather that we seem to be forced to codify principles of inference into statements. To respond by pointing out the difference is not to offer a solution at all. For it is in effect just to say that inference principles ought not to be treated as premises on the grounds that this generates the very puzzle that is to be thus explained. This is to offer no explanation at all. Calling attention to the distinction merely legislates against raising the puzzle.

It will likewise not do to claim that the Tortoise is simply guilty of not recognizing his logical obligations and that Carroll is simply wrong in thinking that the obligations need to be codified before they can have any force (Wisdom 1974). On the above reading, the point is not that the Tortoise might not recognize that A and B are reasons to believe Z. The point is that we seem to be forced to admit that there *must* be yet another reason to believe Z, viz. H₁ (and its infinite company). The Tortoise may well recognize that A and B look like prima facie reasons for Z but in view of the reasoning presented to him, he seems forced to conclude that they are in fact insufficient for him to draw the conclusion, that if he does not accept H₁, the conclusion will not follow. Moreover, the grounds given for such an admission seem perfectly innocuous – they are not that we must make all inference principles explicit. They are as innocuous as the grounds for claiming that a person who believes that abortion is wrong because killing is wrong must also believe that abortion is a killing.

4. So where lies the problem? There are two problems in fact. Both the antecedent and the consequent of (*T*) are ambiguous.

Some verbs (including most of the verbs signifying propositional attitudes) are subject to a scope ambiguity with respect to negation (see e.g. Belnap & Perloff 1990). “ α does not want to

φ ” sometimes means “It is not the case that α wants to φ ” (α lacks a pro-attitude toward φ ing) and sometimes “ α wants not to φ ” (α has a con-attitude toward φ ing). Sometimes the form of words used suggests precisely the opposite reading. “I have no intention of complying with the court’s order” does not merely assert the lack of an intention on the part of the speaker. “I don’t want spinach!” does not merely indicate a lack of said desire; it means “I *want* no spinach”. “I don’t think that you are right” does not merely indicate that the speaker lacks a belief – it is better expressed as “I think that you are wrong.”¹

One crucial step is to note that the antecedent of (*T*) is ambiguous between:

(*T^{Bn}) If one *believed* that P_M is *not* true, one would have no reason to believe that C is true.

and

(*T^{Nb}) If it were *not* the case that one *believed* that P_M is true, one would have no reason to believe that C is true.

Clearly, (*T^{Nb}) is required to show that P_M is missing, that one *must* accept P_M if one is to have a reason to infer the conclusion. Only if (*T^{Nb}) is true does one show that it is necessary to accept that P_M is true in order to infer the conclusion.

This difference is quite fundamental. It is underscored by considering on what basis one is to judge whether or not one has a reason to believe the conclusion. Let us assume that the argument in question has premises P_1, P_2, \dots, P_k and conclusion C . According to (*T^{Bn}), in order to determine whether P_M is missing, we would need to look an *enriched* set of premises $\{P_1, P_2,$

* This paper was written before a paper by Timothy Smiley, “A Tale of Two Tortoises” (*Mind* 104, 1995, 725-736) was brought to my attention. The solution I offer here is indeed the same as Smiley’s, though it is couched in a somewhat different context.

..., $P_k, \neg P_M$. If C does not follow from such an enriched set $\{P_1, P_2, \dots, P_k, \neg P_M\}$ but does follow from $\{P_1, P_2, \dots, P_k, P_M\}$, P_M is claimed to be a missing premise according to (*T^{Bn}). According to (*T^{Nb}), on the other hand, in order to determine whether P_M is missing, we would need to look at the set of premises *as they are* $\{P_1, P_2, \dots, P_k\}$ unenriched, as it were. If C does not follow from the original set of premises $\{P_1, P_2, \dots, P_k\}$ but does follow from $\{P_1, P_2, \dots, P_k, P_M\}$, P_M is claimed to be a missing premise according to (*T^{Nb}).

But not only the antecedent is ambiguous. So is the consequent. There are at least three different ways of allotting the negation:

(T^{Nb}) If it were not the case that one believed that P_M is true, one would *not have* a reason to *believe* that C is true (on the grounds of P_1, P_2, \dots, P_k alone).

(¬T^{Nb}) If it were not the case that one believed that P_M is true, one would *have* a reason *not* to *believe* that C is true (on the grounds of P_1, P_2, \dots, P_k alone).

(=T^{Nb}) If it were not the case that one believed that P_M is true, one would *have* a reason to *believe* that C is *not* true (on the grounds of P_1, P_2, \dots, P_k alone).

Both (¬T^{Nb}) and (=T^{Nb}) require that the lack of the allegedly missing premise provide one with a *positive reason* to either believe or not believe something. They are thus too strong as a test for what is a missing premise. Consider again the argument that abortion is wrong. If one lacked the belief that abortion is a killing while holding the belief that killing is wrong, one would thereby have neither a positive reason to believe that abortion is wrong nor a positive reason not to believe that abortion is wrong. Rather, if one lacked the belief that abortion is a killing, one would simply not have any reason (on the grounds of the belief that killing is wrong alone) to

¹ By contrast, “I don’t know whether you are right” cannot in the same way be expressed as “I know that you are wrong”. In general, most ordinary uses of ‘know’ do not seem to be subject to that same ambiguity. Neither do many uses of ‘see’.

suppose that abortion is wrong. So (T^{Nb}) seems to be the requisite test to determine whether premise P_M is missing in an argument from P_1, P_2, \dots, P_k to C .

5. Let us call (T^{Nb}) the *proper test* for deciding whether a premise is missing in an argument; let us call $(T^{Bn}), (-T^{Bn}), (=T^{Bn})$ the *imposter tests*. It is easy to show that the regress of infinite premises is generated only when using the imposter test.

Let us begin with the latter. Suppose that the imposter test, $(=T^{Bn})$ say, provides the condition for H_1 being a missing premise in the argument from A and B to Z thus

if one believed that H_1 is false, one would have a reason to believe that Z is false.

Surely if one believed that H_1 is false, i.e. that A and B are true while Z is false, this would provide one with a reason to believe that Z is false. So, by the imposter test, one would have to conclude that one could not infer Z from A and B unless one added H_1 to the stock of hypotheses. Since the same reasoning can be applied infinitely, the infinite regress is generated.²

But the imposter test is two equivocations away from the proper test. The antecedent is concerned with what would happen if one actually *had* a belief that H_1 is false not with what would happen if one *lacked* a belief that H_1 is true. The consequent states not that one would *not have* a reason for drawing the conclusion but rather that one would *have* a reason for *not* drawing it. And this makes quite a bit of difference.

² The regress would also be generated were $(-T^{Bn})$ to be used. For it is true that if one *believed* that H_1 is *false*, one would have a *reason not to believe* that Z is true (on the grounds of A, B and not- H_1). Were (T^{Bn}) to be used, the situation is no longer so clear, for it is not clear that if one *believed* that H_1 is *false*, one would *not* have a *reason to believe* that Z is true (on the grounds of A, B and not- H_1). It is arguable that one does too have a reason to believe that Z is true – A and B provide one with such a reason, though one also acquires a different reason to believe that Z is false (not- H_1) provides one with such a reason.

Consider the proper test involving (T^{Nb}) . In order for H_1 to be a missing premise, it would have to be true that:

If it were *not* the case that one *believed* that H_1 is true, one would *not* have a *reason to believe* that Z is true (on the grounds of A and B alone)

But this condition is simply false. One may think that one has a very good reason to believe that Z is true on the basis of A and B even if one lacks any explicit beliefs regarding H_1 .³ But if so then the regress is stopped at the outset.

Indeed, according to (T^{Nb}) , no valid argument will be missing any premises. This will be because without adding any other statement, one will already have a reason to draw the conclusion from the existing premises. This provides a principled reason why principles of inference should not be codified as premises in an argument.

The advantage of this way of thinking about the puzzle is that it explains why the puzzle has been puzzling. Given the ease with which we slide back and forth between the two readings of negated propositional attitudes, we appear to find ourselves in the impossible situation where the addition of H_1 is (on reading (T^{Nb})) and is not (on reading $(\neg T^{Bn})$ or $(\neg T^{Bn})$) required of us.

6. Let us look carefully at the proper test again and compare it to either of the imposter tests. I take it that it is indisputable that in order to claim that the belief that P_M is true is *necessary* to draw conclusion C , it would have to be the case that one could not draw it if one did not have the belief (in the sense of *lacking* it, not merely *having* a *contrary* belief).

³ This coincides with Wisdom's (1974) suggestion that one already has an obligation to draw the conclusion before H_1 is added. What Wisdom does not recognize is that there might really appear to be good reasons to add H_1 if one uses the imposter test.

If we look carefully at these two tests, it will become obvious that there is a sense in which the imposter test is much more useful than the proper test. For it is clear that the imposter test has nice pedagogical qualities that the proper test lacks. After all, the proper test simply amounts to carefully looking at the existing premises to see whether the conclusion follows from them. It adds no new information, however. By contrast, the imposter test does too help in deciding whether the existing premises are sufficient to draw the conclusion. Rather than just resting content with the existing premises, it adds *new information* (the supposition that an allegedly missing premise is false) and thereby lets one decide whether its negation is needed.

Consider the argument that abortion is wrong because killing is wrong. According to the proper test, we must determine whether the one premise (that killing is wrong) is a sufficient reason for drawing the conclusion. In other words, the proper test asks for a determination whether the argument, as it stands (without the addition of any other premises!), is valid. According to the imposter test, we should consider whether one would have a reason for drawing the conclusion if one accepted an additional premise, viz. that abortion is not a killing. It seems unquestionable that the imposter test is more perspicuous. It allows one to imagine what the argument would sound like if voiced by someone who believed that abortion is not a killing. In this way, the imposter tests can steer one's thoughts and make plain connections that one might not have seen before.

7. We can thus say that

Premise P_M is missing in an argument from P_1, P_2, \dots, P_k to C just in case: (a) P_M (conjoined with P_1, P_2, \dots, P_k) provides a reason to believe that C is true and (T^{Nb}) if it were not the case that one believed that P_M is true (i.e. on the grounds of P_1, P_2, \dots, P_k alone), one would not have a reason to believe that C is true.

This is not to say that P_M is the only missing premise, but only that it is *a* missing premise in the argument. Once we accept this characterization of a missing premise, Lewis Carroll's infinite regress is not a threat. I argued that the proper test (T^{Nb}) provides a principled reason for denying that inference rules codified into statements are missing premises of valid arguments. At the same time, it is easy to understand why the puzzle arose in the first place in view of the fact that a closely related imposter test does indeed yield the regress. Moreover, our susceptibility to fall under the spell of the imposter test is increased not only by the fact that we are prone not to distinguish the relevant ambiguities but also by the fact that the imposter test has nice heuristic qualities that the proper test lacks. We must, however, resist the temptation to vote the imposter test to be the test for determining what the missing premises are on the ground of its educational qualities. That would be like subjecting a child to a series of toy advertisements before asking for the toys that he wants. While the proper test might be of no heuristic help in contrast to the imposter test, it stands as a condition that a premise missing from the argument must fulfill. It is as important to realize the heuristic advantages of the imposter test as it is to realize its limitations, of which the Lewis Carroll puzzle should remind us.

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